Studying the consolidation settlement for multi-layered foundation concerning the change in compressibility within the settlement zones

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ARTICLE INFO

ABSTRACT

DOI:10.46223/HCMCOUJS.tech.en.11.1.1450.2021

This paper deals with the approach of predicting the consolidation settlement for a multi-layered foundation concerning the change in the compressibility of the settlement layers. This non-linear problem is preliminarily solved by using the Massachusetts Institute of Technology’s (MIT’s) suggestion of approximation, according to which every two layers are converted into a single layer having an equivalent coefficient of consolidation $C_v$. This paper aims to further the solution by considering the variation of the permeability in the settlement zones of each compressible layer as a change in the compressibility and coefficient of compression. During consolidation, the new value of $k$ is updated, resulting in a new coefficient of consolidation $C_v$ and $m_v$ simultaneously; then compressible layers are combined and converted into either equivalent properties or thickness of consolidation. In case the coefficient of volume compression $m_v$ is kept to be constant, and the change in the compressibility of the settling zone is taken into account as in this study, preliminary results on the settlement of the alternatives indicate that the consolidation settlement will be greater than that of the conventional approach of analysis and the numerical computation by Plaxis model. The results of the comparison between this study and those of the other models _that are assumed to yield the same time-dependent consolidation_ and numerical model Plaxis indicate that the consolidation for multi-layered foundation requires a more rigorous study. This is a more detailed modification for coming closer to a conservative prediction of the time of consolidation and final consolidation settlement.

1. Introduction

Estimating the consolidation settlement of a layered foundation is a complicated task. In multi-layered foundation, and the rate of consolidation settlement for such a multi-layered foundation is a non-linear problem approximately solved by many research works such as: Taking the varied compressibility into account (Miao, Wang, & Kavazanjian, 2008), the varied drainage condition (Bahmanikashkouli & Nezhad, 2012); considering the variation in permeability (Abbasi, Rahimi, Javadi, & Fakher, 2006), etc. A technique of coupling every two compressive layers into one equivalent which is based upon MIT’s suggestion was studied by (Duong, Tu, Nguyen, Le, & Nguyen, 2020).
During the consolidation, there are the settlement zones 1a, 2a, and zones 1b, 2b (Figure 1). The latter ones are called zero-settlement zones as the former zones have the varied void ratio \(e\). In the zone of settlement (i.e., 1a for layer 1 and 2a for layer 2), the compressibility has varied, both in permeability and compressibility. By assuming that the unsettled layer (i.e., 1b or 2b in Figure 1) has no variation in the compressibility so that each compressible layer has two separate sub-layers (e.g., 1a and 2a of layer 1), which the coefficient of consolidation are not the same as the original values. A question is what occurs if the zone of settlement, e.g., 1a or 2a, which the compressibility and the coefficient of consolidation could take into account the change.

This paper aims to further some results studied by (Duong et al., 2020) to investigate the consolidation settlement of the multi-layered compressible foundation, concerning the settlement zones as small zones. This study expects to obtain different results about the consolidation settlement in someway.

2. Theoretical background

2.1. Assumptions

The coefficient of compression \(m_v\) and the compression index \(C_c\) for a soil sample are both determined by compression test in oedometer and their values can be determined as the secant slope of the plot \(e \sim p'\) and slope of the \(e\sim \log(p')\) curves as in Figure 1:

\[ m_v = \frac{\Delta e}{\Delta p'} \frac{1}{(1+e_o)} \]

\[ C_c = \frac{\Delta e}{\Delta \log(p')} \]

Within a specified range of compression effective stress, we can determine the settlement by using either \(m_v\) or the \(C_c\). In this paper, \(C_c\) is given and \(m_v\) will be computed.
The coefficient of consolidation $C_v$ is defined by equation 1, as below:

$$C_v = \frac{k}{m_v \gamma_w}$$

in which $m_v$ is the coefficient of volume compressibility (depends on the range of compression pressure), $k$ is the coefficient of permeability which varies to void ratio, and $\gamma_w$ is the unit weight of pore water. $C_v$ will be constant if there is generally no change in void ratio throughout the compressing layers (for example, zone 1b in Figure 1).

- Each layer has its constant characteristics of compressibility and permeability. The load is fully charged at the start of loading, and the soil is normally consolidated. The condition of equivalence is the two models have the same degree of consolidation.

  - The permeability varies only within the settled zone (i.e., zone 1a or 2a); the coefficient of permeability $k$ would be of 3-order power of the void ratio, or $k \propto e^3$;

$$\frac{k}{k_f} \propto \left(\frac{e}{e_f}\right)^3$$

(2)

The coefficient of compression $m_v$ (kPa$^{-1}$) would change consequently by the formula:

$$m_v = \frac{e_o - e_f}{p'_f - \sigma'_{vo}} \cdot \frac{1}{1 + e_o}$$

(3)

As such, $m_v$ is defined to be the tangent of the compression curve, measuring from compression pressure the initial value to the final value. The sharper the compression curve is, the higher compressibility and vice versa.

- Terzaghi’s theory of 1-D consolidation is still valid within a compression layer. This study will study the 1D consolidation. Within a settling zone (c.f. zone 1a or 2a in Figure 1), the updated coefficient of consolidation will change and keep it to the final stage of consolidation.

- Both the upper and lower boundaries of any layer are previous so that the drainage length would be $d = H_{eq}/2$ in which $H_{eq}$ is the equivalent thickness of the converted two-layer foundation.

- This study considers the variation of permeability as the primary factor of compressibility and the coefficient of compression is kept to be fixed.

### 2.2. Equivalent coefficient and thickness

#### 2.2.1. Equivalent coefficient of consolidation

![Figure 3](image-url). Concept of equivalent coefficient of consolidation $C_v, \text{eq}$

The double-layered foundation is converted to a single layer having the equivalent coefficient of consolidation from equation (4) without changing the thickness of the foundation (Figure 3).
The equivalent coefficient of consolidation is:

$$C_{v,eq} = \frac{\sum_{i=1}^{n} (h_i)^2}{\sum_{i=1}^{n} \left( \frac{h_i}{\sqrt{C_{vi}}} \right)^2}$$  \hspace{1cm} (4)

2.2.2. Equivalent thickness of consolidation foundation

MIT suggests that an equivalent thickness of consolidation foundation can alternatively keep the properties as original values but changes the depth into an equivalent one having the same compressibility (the smaller value of $C_v$ is taken into account) as the equation (5).

$$H_{eq} = h_1 \sqrt{\frac{C_{v1}}{C_{v2}}} + h_2$$  \hspace{1cm} (5)

where $h_1$ is the thickness having the smaller value of the coefficient of consolidation $C_{v1}$. $H_{eq}$ then is the equivalent thickness of the foundation that has the greater value of the coefficient of consolidation $C_{v2}$.

2.3. Suggested procedure for solving the problem

Step 1: Prescribe all the data of effective stress, initial and final values of stress.

Step 2: Based on $C_c$ and the initial void ratio $e_o$, calculate the final void ratio $e$.

Step 3: Calculating the varied coefficient of permeability $k$ by formula (2), then the numerator of the $C_v$ is calculated by the formula (1). As such, the $C_v$ after consolidation could be determined.

Step 4: In case that $m_v$ is kept to be a constant as in the assumption, calculate the updated $C_v$ by formula (1). This $C_v$ will be in the settlement zone.

Step 5: Coupling $C_v$ gained in step 4 to the $C_v$ of the original thickness of the soil layer. The newly converted equivalent coefficient of consolidation will be obtained, namely $C_{v,eq}$. If the equivalent approach is applied, the converted thickness would be $H_{eq}$.

Step 6: Coupling further every two layers to have the converted value of $C_{v,eq}$ or $H_{eq}$, as the schematic diagram in Figure 3 or 4, respectively.

Step 7: The finally converted layer with the equivalent coefficient $C_{v,eq}$ or thickness $H_{eq}$ will be used to compute the consolidation settlement.

3. Model

3.1. Soil profile under consideration

Real soft soil properties at a real site in Thao Dien Ward, District 2, Ho Chi Minh City are
reviewed as in Table 1. The site was subjected to a 1D consolidation with total stress of 32 kPa (nearly equals to 2 meters backfill height over a soft soil foundation).

Table 1

<table>
<thead>
<tr>
<th>No</th>
<th>Borehole</th>
<th>Depth of Sampling (m)</th>
<th>Void ratio ε₀</th>
<th>Coefficient of consolidation Cᵥ (cm²/s) x 10⁻³</th>
<th>Compression Index Cᵥ</th>
<th>Swelling index Cᵣ</th>
<th>Preconsolidation pressure kPa</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BH1</td>
<td>1.8 - 22</td>
<td>2.686</td>
<td>0.328</td>
<td>1.494</td>
<td>0.199</td>
<td>67.3</td>
<td>Clay mud, liquid state.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>22.4 - 24</td>
<td>2.754</td>
<td>0.215</td>
<td>1.322</td>
<td>0.161</td>
<td>33.7</td>
<td>Mud, liquid state.</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>29.8 - 30</td>
<td>0.600</td>
<td>0.457</td>
<td>0.126</td>
<td>0.066</td>
<td>129.0</td>
<td>Brownish Clay, Stiff</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>37.8 - 38</td>
<td>0.506</td>
<td>0.487</td>
<td>0.090</td>
<td>0.016</td>
<td>138.5</td>
<td>Sandy Clay, plastic stiff.</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>39.8 - 40</td>
<td>0.511</td>
<td>0.467</td>
<td>0.102</td>
<td>0.027</td>
<td>137.2</td>
<td>Clayey, semi-stiff.</td>
</tr>
</tbody>
</table>

Source: The researcher’s data analysis

3.2. Numerical model for estimating consolidation settlement

The consolidation of the abovementioned soil foundation was investigated by applying a uniformly distributed loading p= 32kPa as in the schematic diagram in Figure 5. Compressible layers are enumerated by Layer 1, 3, 4, 5, and 6. Layer 3 and 6 are very stiff ones, so they are incompressible layers and their settlement are negligible, not be mentioned in this study.

![Figure 5](image-url)

Figure 5. Soil layers for the problem under consideration

Some windows of the input data for the numerical model are shown in Figure 6.
Figure 6. Data for the numerical model of the problem

4. Results

4.1. Consolidation settlement

The consolidation settlement can be calculated by the conventional procedure and the numerical model. Under the overburden pressure $p_o$ (e.g., layer 1, $p_o=43.4kPa$) and additional pressure $\Delta p = 32kPa$, total settlement calculated and described in Table 2 below:

Table 2
Consolidation settlement

<table>
<thead>
<tr>
<th>Layer</th>
<th>Level</th>
<th>Thickness (m)</th>
<th>Settlement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ele. -1.8 to Ele. 22</td>
<td>22.4</td>
<td>139.34</td>
</tr>
<tr>
<td>4</td>
<td>Ele. -29.8 to Ele. -32</td>
<td>2.2</td>
<td>11.2</td>
</tr>
<tr>
<td>5</td>
<td>Ele. -33.8 to -40.0</td>
<td>6.2</td>
<td>22.8</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>173.34 mm</td>
</tr>
</tbody>
</table>

Source: Data analysis result of the research

The time-dependent settlement for the conventional procedure (i.e., without taking the varied compressibility or the settlement zones into account) is calculated and plotted as in Figure
7b. The time-dependent settlement curve obtained by the numerical model (i.e., Plaxis 2D model) is shown in Figure 7a. There is a good agreement between these two curves.

Figure 7. A comparison between the results of a numerical model and conventional approach

4.2. Equivalent coefficient of consolidation and thickness

The equivalent coefficient $C_{v,eq}$ and the equivalent thickness $H_{eq}$ are calculated by the formula (4), (5), and steps for solving in the abovementioned sub-item 2.3. Taking the compressibility of settlement zones into account and coupling $C_v$ of the zone 1a and 1b altogether, an equivalent and modified value for $C_v$ is found. According to the soil layers described in Table 1 and three compressible layers in Table 2, converted values of the soil properties are calculated and shown in Table 3 (red color numbers in italics).

Table 3
Converted values of equivalent coefficient of consolidation $C_{v,eq}$ and thickness $H_{eq}$

<table>
<thead>
<tr>
<th>Layer</th>
<th>Parameter</th>
<th>Case 1: Equivalent coefficient $C_{v,eq}$</th>
<th>Case 2: Equivalent thickness $H_{eq}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$C_{v,eq}$</td>
<td>$C_{v,eq}$</td>
</tr>
<tr>
<td>1</td>
<td>Thickness (m)</td>
<td>22.4</td>
<td>3.26e-4</td>
</tr>
<tr>
<td></td>
<td>$C_v$ (cm$^2$/s)</td>
<td>3.28e-4</td>
<td>3.27e-4</td>
</tr>
<tr>
<td></td>
<td>This study</td>
<td>3.273e-4</td>
<td>3.273e-4</td>
</tr>
<tr>
<td>4</td>
<td>Thickness (m)</td>
<td>2.2</td>
<td>3.32e-4</td>
</tr>
<tr>
<td></td>
<td>$C_v$ (cm$^2$/s)</td>
<td>3.32e-4</td>
<td>3.27e-4</td>
</tr>
<tr>
<td></td>
<td>This study</td>
<td>3.11e-4</td>
<td>3.11e-4</td>
</tr>
<tr>
<td>5</td>
<td>Thickness (m)</td>
<td>6.20</td>
<td>3.32e-4</td>
</tr>
<tr>
<td></td>
<td>$C_v$ (cm$^2$/s)</td>
<td>4.77e-4</td>
<td>4.77e-4</td>
</tr>
</tbody>
</table>

Source: Data analysis result of the research

4.3. Results of the time-dependent settlement for this study

According to the results of the abovementioned converted values of $C_{v,eq}$ and $H_{eq}$ in Table 3, the time-dependent settlement could be computed by Terzaghi’s theory of 1D consolidation, results are described in Table 4. A finite element model is created with Plaxis 2D V.8.5 as in Figure 5, with Mohr-Coulomb model of consolidation analysis for comparison purpose. All the results are tabulated to be compared with each other and shown in Table 4.
Table 4
Consolidation settlement up to 500 days (settlement in cm)

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Settlement by using $C_{v,eq}$</th>
<th>This study (adjusted $C_{v,eq}$)</th>
<th>Settlement by using $H_{eq}$</th>
<th>This study (adjusted $H_{eq}$)</th>
<th>Settlement by Numerical model (Plaxis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0.0</td>
<td>0.000</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>4.218</td>
<td>6.8</td>
<td>4.972</td>
<td>7.8</td>
<td>4.5</td>
</tr>
<tr>
<td>200</td>
<td>7.607</td>
<td>9.6</td>
<td>8.109</td>
<td>11.0</td>
<td>7.5</td>
</tr>
<tr>
<td>300</td>
<td>9.013</td>
<td>11.8</td>
<td>9.766</td>
<td>13.4</td>
<td>9.7</td>
</tr>
<tr>
<td>400</td>
<td>10.419</td>
<td>13.6</td>
<td>11.424</td>
<td>15.5</td>
<td>11.5</td>
</tr>
<tr>
<td>500</td>
<td>11.825</td>
<td>15.2</td>
<td>13.081</td>
<td>17.3</td>
<td>13.6</td>
</tr>
</tbody>
</table>

Source: Data analysis result of the research

The abovementioned results are plotted to have a clearer comparison, as shown in Figure 8.

Figure 8. Results of time-dependent settlement calculated by the equivalent consolidation, with and without concerning the compressibility in the settlement zones

4.4. Discussion

- The soil in the real site is an overconsolidated one; lightly overconsolidated one (OverConsolidated Ratio OCR=1.2 > 1), so that the most of consolidation settlement for the upper layer (i.e., Layer 1) is compensated by a process of re-loading from the previously loaded pressure $p_c=50.5$ kPa to the current overburden pressure 43.4 kPa; then comes the new process of settlement, from the overburden pressure to the imposed load $p=32$ kPa. So the settlement is small. This study is a further step for clarifying the point that if the variation in the compressibility of settlement layers is taken into account, the time of consolidation could be significantly different than the case of disregarding the change of compressibility. The compressibility is characterized by the void ratio and the modulus of compressibility, as below:

$$E_{i-j} = \frac{\beta}{m_v}$$  \hspace{1cm} (6)

in which $m_v$ is defined by formula (3). This quantity should be determined by laboratory test instead of computing from dimensionless value $C_c$. The correlation between $m_v$ and $e$ may tentatively be determined by specific laboratory tests;
• The coefficient of compression, $m_v$, in kPa$^{-1}$, should not be calculated via the dimensionless value of $C_c$. It should be determined by compression tests in the laboratory. Unlike the correlation between the permeability $k$ and void ratio as described in the expression (2), there is no literature review on the proportion between the $m_v$ and $e$. These are the main reasons for explaining why this study preliminarily only considers the variation of $C_v$ to the numerator of the formula (1).

• The variation of compressibility is quantified by the third-order power of the void ratio $e$, but the correlation depends on the soil kind; some soil expresses a parabolic relationship (Arora, 2004). In the other words, $k \propto e^3$ results in the linear correlation $C_v \propto k$. This should be checked by a laboratory in oedometer tests, to confirm this important proportioning correlation between the permeability $k$ and the void ratio $e$.

\[ \text{Fig. 9. Variation of } k \text{ with } e^2, \frac{e^2}{1+e} \text{ and } \frac{e^3}{1+e}. \]

As for various kinds of soil, the linear correlations between $k$ and $e^2$ and $e^3$ (c.f Figure 9, in which $1+e$ stands for the specific volume), so that this study uses the third-order power correlation (i.e., $k \propto e^3 \propto \text{const} \times e^2$).

• The results by converting into an equivalent thickness $H_{eq}$ (i.e., keep the coefficient of consolidation being constant) appears to match with those of the numerical model. And the results of the time-dependent consolidation in which the settlement zones are taken into account are not longer as predicted. As such, Both the coefficient of permeability $k$ and the coefficient of compression $m_v$ should be considered simultaneously.

• As the rule of thumb, results should be compared to those of site observation and measurements to determine the change in compressibility with more accuracy.

5. Conclusions

Consolidation settlement could be estimated by using an equivalent coefficient of consolidation or equivalent thickness of compressible layers according to MIT’s approximation. For a supplementary study, the effect of the change in permeability on the consolidation settlement is taken into account first, before having sufficient information of the coefficient of volume compression $m_v$ that is always measured in the laboratory, not by computing from $C_c$. Every two compressible layers could be coupled and converted to a single modified layer, and so on for other layers in the foundation. This study suggests that every two layers will be coupled to convert them into one single and the settlement zone may be viewed as a modified layer having varied
compressibility. From the change in the void ratios, the variation of the coefficient of permeability results in the change in the coefficient of consolidation $C_v$. The approach in this study predicts a more conservative result of the consolidation settlement. So the approach of coupling layers might be beneficial in practical application, especially in preliminary estimation. This idea of this study which takes the settlement zones into account should be studied more both in theoretical and experimental aspects for predicting the time-dependent settlement with more accuracy in the long term.

References


