MATCHING ALGORITHM OF THE AUCTIONS ON EBAY

By Nguyen Van Phuong¹

This paper explores eBay auction properties that match buyers and sellers and generates millions of sales every month. eBay's auction is now a well known mechanism designed to make buyers and sellers feel comfortable doing business without meeting each other. In a theoretical point of view, the current matching algorithm has not solved the online auction problems because the main conditions of agents' preferences do not satisfy when bidders are unobservable and a set of bidders is not identified. Therefore, we construct a new simplified model of matching with a given object for sale to form a seller-bidder pair to overcome the online auction issues. Specially, our model may extend for the matching mechanism of the job market.

I. Introduction

Auctions have become a popular tool in various e-commerce settings. The number of auctions conducted on the internet has increased at a tremendous rate because they are convenient for sellers and buyers not only to trade at lower transaction costs but also they are alternatives to traditional bargaining methods. One of the earliest and best known internet auctions is run by eBay, which matches buyers and sellers and facilitates millions of sales every month. Gross merchandise volume, or the total value of all successfully sold items on eBay's sites, was \$ 10.8 billion in 2005, compared with \$ 8.3 billion one year ago (The WSJ, Oct. 20/2005). Up to the third-quarter of 2010, eBay said total payment volume jumped to 26 percent to \$22.37 billion compared with the same period of 2009 (AP Technology Writer, Oct/20/2010).

The reputation of eBay in conventional retail markets has caused traditional retailers and firms such as Sears, Walt Disney, IBM,

Xerox and many others to trade on eBay as well in order to capitalize on their brand names (Hof, 2003). Information on the reliability provided by eBay of buyers and sellers on eBay is measured by the rating and trading historical data. Under the rating system, any trader can report its experience with another trader rating the other as positive (+1), neutral (0) or negative (-1). The cumulative score is shown on the site as Feedback Rating for that trader (Lucking-Reiley et al, 2007).

eBay's auction is now a well known mechanism designed to make buyers and sellers feel comfortable performing transactions, exchanging cash for goods without meeting each other. Sellers have several auction options, such as payment methods, the length of the auction (from 3 to 10 days), the starting price and reserve price or a "Buy it now" option that allows buyers to purchase immediately at a pre-specified price.

There is a huge literature, both theoretical and empirical, on auctions, but little of it applies matching theory to the study of online auctions. The theory of auctions and competitive bidding by Milgrom and Weber (1982) describes common auction forms such as the English auction, the Dutch auction, the first price sealed bid auction and the second price sealed bid auction. Riley and Samuelson (1981) address the issue of seller's choice of auction design by considering the choice of a reserve price.

Matching algorithms have played an important role in practical applications. Gale and Shapley (1962) introduce a matching mechanism for the college admission problem. Roth and Peranson (1999) develop a two-sided matching algorithm used to match 20,000 doctors per year to medical

¹ Post & Telecommunications Institute and Technology at Ho Chi Minh City

residency programs. Abdulkadiroglu and Sommez (2003) introduce a variation of the same algorithm for use by school choice programs. Similar procedures are being evaluated by several large city school districts. Hatfield and Milgrom (2005) explore a model of matching with contracts which incorporates, as special cases, the college admission problem, the Kelso-Crawford labor market matching model and ascending package auctions.

A number of empirical studies of online auctions have also been completed. Lucking-Reiley (1999 and 2005) implements several empirical studies of online auctions by using an initial data set of 20,000 auctions. The regression analysis concludes that, first, a seller's feedback rating has a measurable effect on her auction prices. Second, minimum bids and reserve prices have positive effects on the final auction price. Third, a seller's choice to have her auction last for a longer period of days significantly increases the auction price on average. Resnick and Zeckhauser (2002) find that the effect of seller reputation has predicted positive impact on seller revenues when proper experimental controls are imposed. Anderson et al (2007) use 1177 samples of eBay auctions and conclude that different types of sellers pursue systematically different strategies for how their items are offered. For example, more highly rated sellers are more likely to provide clearly detailed product information, as well as secure payment options. Song (2004) develops and applies new methods for analyzing auctions in which the number of potential bidders is unknown. This research was motivated by Internet auctions and is useful of other ascending auctions as well.

Buyers and sellers are heterogeneous and search for the right trading partner. Each buyer's choices are independent of those of other buyers but his behavior does affect his competitors. Bidders who increase the current winning bid cause either to increase

their bids or else withdraw from the auction. The seller's expected profit increases with the number of participating bidders. Those characteristics will be explored in the matching section.

In the online auction, bidders are unobservable. Therefore, we cannot determine a set of bidders. As a result, each seller cannot prefer over the bidders. Moreover, each bidder may or may not prefer over the sellers because he can bid the same object from different sellers at the time t . In essence, the general matching algorithm is not applied for solving the online auction problem because the main conditions of agent's preferences do not satisfy. Therefore, this paper introduces a new simplified model of matching with a given object for sale to form a seller-bidder pair. The matching process creates a set of outcomes. An outcome includes an object for sale, a seller, a bidder and a bid price at the time t . In other words, a set of outcomes is a result of the matching process. The final outcome will determine the winner of the auction and a contract to trade is automatically and immediately signed at the end time of the auction if his bid price is at least equal to a seller's reserve price. Otherwise, there is neither contract nor winner. If no bid is placed, then there is no outcome. The number of outcomes is equal to the number of bidding times. In our outcome model is similar to the contract model by Hatfield and Milgrom, i.e, it also satisfies the law of aggregate demand and outcome substitute condition. In other words, a seller prefers the latest outcome to the previous ones.

Our model can explore in the job market when employers and employees throughout a broker company or a head hunter company. A head hunter company plays a role as a matchmaker. Given available positions from the employers, a matchmaker matches each employee who applies for such an available position

with each employer. The outcome of the matching algorithm includes an available position, an employer, an employee and an application profile. Like the online auction, a matchmaker also informs the termination time to receive application form. However, the final outcome is not so important element like in the online auction.

The rest of paper is structured as follows. Section 2 introduces the eBay auction rules. Section 3 explains restrictions placed upon the model. Section 4 explores the matching process and its outcomes in this market. Section 5 extends the model applied for the job market. Section 6 concludes with a summary and suggestions for further research.

II. eBay Auction Rules

This section explains briefly the eBay auction mechanism. An eBay auction starts as soon as a seller registers all information of goods for sale. The seller provides information on the product offered for sale, such as a description or a picture, terms of payment and shipping, and the duration of the auction (either 3, 4, 5, 7 or 10 days). The seller can also choose a reserve price or a "Buy it now" price. With a low starting price a seller can attract buyers to bid. A reserve price can stimulate bidding while preserving the seller's right not to sell below a minimum price. When a seller wants to sell immediately to any buyer who values the item at least the reservation price, the "Buy it now" option is the best choice.

In this market, seller S_i offers the good i with a private seller information. However, after placing a bid, eBay informs whether a bidder exceeds seller's reserve price or not.

Potential buyers can bid on any good posted for sale on eBay. The history of bids is available to them. The auction ends at the pre-specified time. The good is sold to the winner at the highest bid price.

Finally, when a trade is completed both the buyers and sellers can use eBay

to comment upon and rate each other. This allows both buyers and sellers to develop and maintain reputations.

Every user on eBay has to register and to agree to **eBay User Agreement and Privacy Policy** before participating in an auction. All eBay auctions use an open, ascending-bid (English) format that significantly differ from the traditional auction's ascending bid format in two ways. First, there is a fixed ending time instead of a "going-going gone" ending rule. Second, eBay uses a proxy bidding system. A bidder is asked to submit a maximum bid or a cutoff price, instead of his instant bid amount. The proxy bidding system then will issue a proxy bid equal only to the minimum increment over the next highest bid (Song, 2004). We will consider the effects of these two characteristics matching algorithm in section 4.

III. Restriction on the Model

Each buyer can bid on any object offered for sale. Each seller can offer either a single good or a bundle of goods for auction. In the case a seller offers a bundle of goods for sale, we suppose that the bid prices of each good in a bundle are uncorrelated. In our model, a buyer is free to bid more than one goods offered by a single seller but his bid prices are totally uncorrelated with each other. In other words, if a buyer wants to buy both of goods from the same seller, he must be placed the highest price for each good independently. This assumption guarantees that not only outcomes but also sets of outcomes are completely independent of each other. Without this assumption, the analysis becomes more complicated because it is too difficult to determine the winner for each good.

Note also that online auctions differ from many other auctions in that no agent can stop the online auction before the specified termination time even though both the seller and some buyer might prefer to trade at an earlier time.

IV. Matching Mechanism

Notation

Let S_i ($i \in \{1, \dots, n\}$) denote seller i offering good i for sale. Although, a good i can be offered by S_i , as our assumption, we suppose that there are two different objectives for sale. Let r_{si} denote seller i 's choice of reserve price. If seller i does not set the reserve price then $r_{si} = 0$ and the good will be sold for the bidder who offers the highest price at the end time of the auction. A seller can also lower his reserve price after the goods is listed. eBay notifies all buyers in the history set of the auction of any such change.

Let B_i be a set of buyers participating in the online auction for given a good i . Let h_i be the history of bids made on the object offered for sale by seller i , $h_i = (p_i^1, \dots, p_i^k)$ if $k \geq 1$ bids are made, otherwise $h_i = \emptyset$. The rules of the online auction ensure that the elements of h_i are strictly increasing. The number of potential buyers is not known and is unobservable. However, eBay allows the seller to observe the history of bids and who places a bid. Thus, attached to each bid p_i^t is eBay ID, date of bid and email address of the person who bids p_i^t . p_i^1 is the initial bid. p_i^k is the highest bid price. When a new bid price is placed, eBay notifies all agents of the highest bid at the current time t . The current highest is notified of his position and whether his bid has met the reserve price.

Intuitively, the number of buyers is not observable but it must be countable and finite at the termination time. If it is infinite then p_i^k is very huge. This is unrealistic since every buyer has his reserve price and no one wants to bid higher than his reserve price. Let r_{bi} denote a reserve price of a buyer who attends the auction for given a good i .

As we have mentioned that the general conditions for two-sided agents matching algorithm do not satisfy in the online auction market. A set of buyers cannot determine before the auction and each seller

has no preferences over a set of buyers. But each seller has strict preferences over the bid prices and buyers and sellers are heterogeneous and search for the right trading partner. Moreover, if the seller's reserve price is met, then both agents have achieved the agreement for the object to be traded. Therefore, it is necessary to develop a new matching mechanism to solve the online auction problem.

Definition: Matching with a given object for sale

The matching process with a given object for sale is to determine a set of outcomes. An outcome is a result of the matching process whenever a bid is placed for a given object for sale during the fixed ending time. A set of outcomes accumulates all outcomes during the interval time of the auction.

Let $O^j(g_i, S_i, B_i, p_i^j)$ is denoted an outcome of matching algorithm for given good i from seller i and a buyer who submit p_i^j at the time j . ($j \in \{1, \dots, k\}$). Therefore, a set of outcomes is a finite lattice subjected to p_i^j .

From our notation, a set of outcomes includes k elements and the final outcome is $O^k(g_i, S_i, B_i, P_i^k)$. The final outcome is playing a very important role in determining whether the auction is successful or not. If P_i^k is at least equal to r_{si} , then the auction is successful vice versa.

The final outcome is a stable outcome and a Pareto optimal in this market if P_i^k is at least equal to r_{si} , i.e, outcomes are satisfied by the substitute condition because a seller prefers the latest outcome to any other outcomes. She has preferences over the set of outcomes as follows.

$$O^k \succ O^{k-1} \succ \dots \succ O^1$$

Two outcomes can have the same seller-buyer pair since they are two different bid prices thus two outcomes are independent of each other and a set of outcomes is independent and strictly increases.

The seller's expected profit increases with the number of outcomes. Because, a set of outcomes is finite lattice, the more increase in outcomes, the larger gap between P_i^k and r_{s_i} is. Here, the value of P_i^k cannot exceed the buyer's reserve price.

Each buyer's choices are independent from those of other buyers but his behavior does affect his competitors. A buyer who increases the current winning bid causes either to increase competitors' bids or else withdraw from the auction. In other words, a bidder will continue the auction if her/his reserve price is still above the current price, otherwise she/he has to withdraw.

The outcomes in our model are similar to the contracts of Hatfield and Milgrom's model (2005). They are also satisfied by two main conditions, substitute and law of aggregate demand conditions. In particular, in our model the lower seller's reserve price, the more number of outcomes is (law of aggregate demand condition satisfied) and a seller most prefers the final outcome to any other ones (substitute condition satisfied). However, the outcomes in our model also have some particular things. First, if P_i^k is at least equal to r_{s_i} , there exists only one stable outcome at the termination time (the final outcome). Meanwhile there may have more than one stable contract and the time element is not an important one in Milgrom's model. Second, in the generally matching algorithm, based on both agents' preferences, both agents such as doctors and hospitals, man and woman or workers and firms are matched to form each pair, but in our model based on a given good for sale, the matching process is to determine a set of outcomes with all feasible seller-buyer pairs, and then the final outcome with a seller-buyer pair to sign the contract if a seller's reserve price is met. Finally, each seller can offer more than one object for sale and each buyer can also bid many objects at the same time under the condition in which all objects for sale are independent of each other and satisfy our assumption.

eBay is playing important role as a central intermediary in matching to create a set of outcomes. On eBay auction, a buyer is a person who could belong to history set (H) or completely different from H at the time (t) and she/he and a seller S_i are matched to create an outcome. A matching is a rule that specifies all such possible outcomes linked into a set. At the deadline time, all bidders participating in the auction are already included the set of outcomes, therefore the final outcome of the online auction is an endogenous matching in which the auction contract is accomplished by a seller-buyer pair if the seller's reserve price is met.

Similarly to other markets, the online auction market has also some main characteristics as follows. First, if the auction is successful then there is only one equilibrium price satisfy Pareto efficiency, i.e., any contract with lower price than p_i^k makes at least one buyer better off, but making at least one seller worse off. Second, there is always a subset of the buyers who lose in the auction. The more bidders participating in the auction is the larger size of the unmatched subset. However, this market has some particular situations making it be different from the other markets.

First, one of the most important factors making matching results on eBay be much more different from the other markets is that the game can only stop at the fixed end time (k) of auction. Therefore, during the interval time $[1, k]$, a new bid price will completely depend on the current bid price. i.e., every bidder offers the price totally based on her/his previous competitors. In other words, the game can not finish at the time t, even though an outcome includes a seller and a buyer who are able to be matched to form a pair and the potential contract may be signed. Meanwhile, in the other matching algorithm such as men and women in the marriage market, workers and employers in the labor market or students and college in the admission market, if one of the agents rejects the match, the other

agent is indifferent between accepting and rejecting. Furthermore, if both players are matched and got the optimal choice, then the contracts will be signed between the couple, no one wants to change their partner and the matching process is finished. In other words, the game will immediately end when two agents are matched to form a pair.

Second, the final outcome plays an important role in determining the winner of the auction. This property is completely different from the other markets. For instance, in the financial market, an outcome is stable if there is no intermediary-firm pair that would be strictly better off than the initial outcome (Dam and Perez-Castrillo, 2006). Obviously, the initial outcome plays a significant role in the financial market. Moreover, this property contrasts totally in many normal markets in which the expression "First Come First Serve" is used as a service policy whereby the requests of customers are attended to in the order at they arrived, without other preferences (Roth, Alvin and Marilda Sotomayor, 1990). The policy can be employed when processing sales orders or in determining restaurant seating, for examples. This natural difference would be expressed under the phrase "Last Come First Serve" used for the online auction on eBay. The final outcome has impacted on bidders' strategy. In particular, bidders are intensive to bid late. As a result, many bidders could be lost a chance to place their true price due to the expiration time.

Third, given an object for sale, the matching process is performed totally independent among individual agents. In other words, matchmaker performs the matching algorithm for each good with bidders attending the auction of that good independently even though a seller can offer many goods for auctioning and a bidder can also participate in many auctions. This matching mechanism is simpler than that of other markets. The sellers and buyers do not need to meet each other during the auction

and when the final outcome determines the winner to form a pair and the contract will be signed. The final bid price p_i^k and the private reserve price r_i are the most important factors for the successful auction on eBay.

Now we consider whether each agent can get benefit to state his or her true preferences to the matchmaker or not. In the marriage market, they have proved that, at least sometime, "honesty" may not be the best policy (Roth and Sotomayor, 1990). In the online auction, both agents can also misrepresent his or her preferences to the matchmaker because a seller can change the reserve price and a bidder can also place other bid prices during the interval time. In reality, a buyer always wants to place the bid prices as low as possible but a seller wants to sell the goods with a price as high as possible. Hence, a bidder often keeps his true price until the last minute to avoid bidding overvalued price, otherwise, a seller sometimes sets the high reserve price to extract consumer surplus as much as possible. However, sellers' strategy should be changed as soon as possible, if no bid price is placed higher the reserve price for a while. In this case, a seller has to adjust the private reserve price equal to her marginal cost. Therefore, at the beginning time of auction, the misrepresentative strategies are seemed to be better off for both agents. However, in the nearly end time of auction, both agents should represent the true price to make sure the items to be traded at the fixed deadline because no one can change the price p_i^k and r_i at the time k . If one of them misrepresents his or her offering price then he or she might not have a chance to represent her true price at the last minute, even though her true price is higher than p_i^k . As a result, she will not be the winner in the auction. Hence, the fixed deadline property has played an important role in determining when they need to submit the true price to make sure a successful auction. Moreover, placing true price will help the bidder

saving more time during participating in the auction if he actually likes that good.

Social Welfare

Intuitively, a seller always wants to extract as much as possible a buyer's surplus. In contrast, a buyer always wants to pay as low as possible. In the online auction, each buyer's surplus from the successful auction is $r_{bi} - p_i^k$ and each seller's surplus $p_i^k - r_{si}$ therefore, social welfare for this market is equal to

$$\sum_{i=1}^n (r_{bi} - p_i^k) + \sum_{i=1}^n (p_i^k - r_{si}) = \sum_{i=1}^n (r_{bi} - r_{si})$$

V. Model with the Job Market

Our model may explore in the job market when employers and employees throughout a broker company or a head hunter company to recruit or look for a job. The head hunter company plays a role as a matchmaker. Given available positions from the employers, the matchmaker matches each employee who applies for such an available position with each employer. The outcome of the matching algorithm includes an available position, an employer, an employee and an application profile. Like the online auction, the matchmaker also informs the termination time to receive application form. However, the final outcome is not so important element like in the online auction because the first comers may fill up all available positions. Furthermore, a set of outcomes in this market does not necessary to be a finite lattice and there may exist more than one stable outcome, any outcome can lead to the successful employment contract. To sum up, our model can flexible to solve a particular market if our assumption is satisfied.

VI. Conclusion

This paper introduces a simple model of matching with a given object for sale in eBay auction market to create a set of outcomes in which the final outcome is

playing an important role in determining the successful auction. The fixed deadline is a significant element in planning agents' strategies and performing the matching process. In other words, both agents induce their own policies to determine the suitable time when they can misrepresent the preferences and when they should offers the true prices. The final outcome in this market contrasts with many normal markets that can be summarized in a phrase "LAST COME FIRST SERVE" instead of "FIRST COME FIRST SERVE". The model may extend the job market when employers and employees throughout the head hunter companies to recruit and look for a job. Finally, this model should be developed to explore the general case in which there exist at least two objects for sale dependent of each other.

VII. References

- Abdulkadiroglu, Atila and Tayfun Sönmez. (2003). School Choice: A Mechanism Design Approach. *The American Economic Review* , 729-747.
- Anderson, T. Steven, Daniel Friedman, Garrett Milam, and Nirvikar Singh. (2007). Seller Strategies on eBay: Does size matter? *International Journal of Electronic Business* , 643-669.
- Dam, Kaniska and David Perez-Castrillo. (2006). The Principal-Agent Matching Market. *The Berkeley Electronic Journal of Theoretical Economics* .
- Gale, David and Lloyd Shapley. (1962). College Admissions and the Stability of Marriage. *American Mathematical Monthly* , 9-15.
- Hatfield, W. John and Paul R. Milgrom. (2005). Matching with Contracts. *The American Economic Review* , 913-935.
- Hof, R. (2003). The Ebay economy. *Businessweek (August 25)* , 125-128.
- Lucking-Reilly, David, Doug Bryan, Naghi Prasad, and Daniel Reeves. (2007).

- Pennies from Ebay: the Determinants of Price in Online Auctions. *The Journal of Industrial Economics* , 223-233.
- Milgrom, R. Paul and Robbert J. Webber. (1982). A Theoary of Auctions and Competitive Bidding. *Econometrica* , 1089-1122.
- Resnick, Paul and Richard Zeckhauser. (2002). Trust among strangers in internet transactions: Empirical analysis of eBay' s reputation system. *Advances in Applied Microeconomics* , 127-157.
- Riley, G. John and William F. Samuelson. (1981). Optimal Auctions. *The American Economic Review* , 381-392.
- Roth, Alvin and Marilda Sotomayor. (1990). Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis. *Cambridge University Press*.
- Roth, E. Alvin and Elliott Paranson. (1999). The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design. *The American Economic Review* , 748-780.
- Song, U. (2004). Nonparametric Estimation of an eBay Auction Model with an Unknown Number of Bidders. *University of British Columbia*.